2226 – 5016M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2026

Mathematics: analysis and approaches

Higher level

Paper 2

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite student errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some students may be advantaged for that specific question item, it is likely that these students will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that students may be using an incorrect value.

- If the student’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when students fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a student makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a student incorrectly copies values or information from the question, this is a mis-read (***MR***). A student should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of students' own work does **not** constitute a misread, it is an error.
- If a student uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Students will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the student work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however students may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Students are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a student has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10 Presentation of student work

Crossed out work: If a student has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the student indicates that they would like the work to be marked.

More than one solution: Where a student offers two or more different answers to the same question, an examiner should only mark the first response unless the student indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) attempt to use cumulative frequency of 26 or 26.5 e.g. line drawn on graph ***(M1)***

170 cm (accept 170 to 171 inclusive) ***A1***

[2 marks]

(b) (i) recognising the need to find the difference in the cumulative frequencies of $170, 175$ or $175, 180$ (could be seen on graph) ***(M1)***

$42-25$ or $52-42$

$p=17,\ q=10$ ***A1***

(ii) attempt to use midpoints (may be seen in table) ***(M1)***

$157.5, 162.5, 167.5, 172.5, 177.5$

attempt to multiply their values by frequencies and add ***(M1)***

$3(157.5)+8(162.5)+14(167.5)+17(172.5)+10(177.5)\left(=8825\right)$

estimate of mean height $=\dfrac{8825}{52}=169.711...$

$=169.7$ cm ***A1***

Note: Answer must be given to 4 significant figures for ***A1***.

[5 marks]

Total [7 marks]

2. METHOD 1

valid attempt to find at least one critical value ***(M1)***

$-0.618033...$ and $1.61803...$ OR $\frac{1-\sqrt{5}}{2}$ and $\frac{1+\sqrt{5}}{2}$ ***(A1)***

recognition that f is decreasing when $f'(x) < 0$ ***(R1)***

$-0.618 < x < 1.62 \quad \left(\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}\right)$ ***A1***

METHOD 2

$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - x(+c)$ ***(A1)***

attempt to find the x-coordinate of at least one turning point of f ***(M1)***

$-0.618033...$ and $1.61803...$ OR $\frac{1-\sqrt{5}}{2}$ and $\frac{1+\sqrt{5}}{2}$ ***(A1)***

$-0.618 < x < 1.62 \quad \left(\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}\right)$ ***A1***

Total [4 marks]

3. (a) $a = 23$ ***A1***

$b = 20$ ***A1***

attempt to use $\frac{360}{c} = 4$ ***(M1)***

$c = 90$ ***A1***

Note: Award these marks independently.

[4 marks]

(b) **EITHER**

attempt to substitute a valid point into $h = 23 - 20\sin\left(90\left(t + d\right)\right)^\circ$ ***(M1)***

$3 = 23 - 20\sin\left(90\left(0 + d\right)\right)^\circ$ or $23 = 23 - 20\sin\left(90\left(1 + d\right)\right)^\circ$ or $43 = 23 - 20\sin\left(90\left(2 + d\right)\right)^\circ$

OR

recognition that the sine curve is translated horizontally by 1 ***(M1)***

$h\left(t\right) = 23 - 20\sin\left(90\left(t + 1\right)\right)^\circ$

THEN

$d = 1$ ***A1***

[2 marks]
Total [6 marks]

4. (a) A at $(-0.700108..., 0.468564...)$, B at $(0.700108..., -0.468564...)$

$A(-0.700, 0.469)$ $B(0.700, -0.469)$ (accept -0.7 and 0.7) ***A1A1***

Note: Award ***A1A0*** for both correct x-values or both correct y-values.

[2 marks]

(b) **METHOD 1**

valid attempt to find equation of (AB) ***(M1)***

gradient $= -\dfrac{0.468564...}{0.700108...}$

$= -0.669274...$

equation $y - 0.468564... = -0.669274...(x + 0.700108...)$ or $y = -0.669274...x$ ***(A1)***

attempt to subtract definite integrals to find at least one enclosed area ***(M1)***

$\int_{-0.700108...}^{0} (g(x) - (-0.669274...x))\mathrm{d}x \left(= 0.0406871...\right)$ or

$\int_{0}^{0.700108...} ((-0.669274...x) - g(x))\mathrm{d}x \left(= 0.0406871...\right)$ ***(A1)***

$= 0.0406871...$ or $0.0406872\ldots$

$= 0.0407$ ***A1***

METHOD 2

area between g and x-axis in quadrant 4.

$= -\int_{0}^{0.700108...} g(x)\mathrm{d}x = 0.204709...$ (seen anywhere) ***(A1)***

valid attempt to find area between (AB) and x-axis in quadrant 4 using a triangle ***(M1)***

$\dfrac{0.700108... \times 0.468564...}{2} = 0.164023...$ ***(A1)***

attempt to subtract areas ***(M1)***

$0.2047209... - 0.164023...$

$= 0.0406871...$ or $0.0406872...$

$= 0.0407$ ***A1***

[5 marks]

Total [7 marks]

5. attempt to use inverse normal with the standard normal distribution ***(M1)***

$\frac{171-\mu}{\sigma} = \text{InvNorm}(0.6) = 0.253347...$ (seen anywhere) ***(A1)***

recognition of conditional probability in context ***(M1)***

$\text{P}\left(H > 175 \middle| H > 171\right)\left(= \frac{\text{P}\left(H > 175\right)}{\text{P}\left(H > 171\right)} = 0.35\right)$

$\text{P}\left(H > 175\right) = 0.14$ ***(A1)***

$\frac{175-\mu}{\sigma} = \text{InvNorm}(0.86) = 1.08031...$ ***(A1)***

attempt to solve simultaneously ***(M1)***

$\sigma = 4.83692..., \mu = 169.774...$

$\sigma = 4.84, \mu = 170$ ***A1***

Total [7 marks]

6. METHOD 1

$P = \dfrac{135}{V}$

attempt to use chain rule ***(M1)***

$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{\mathrm{d}P}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t}$ OR $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}P} \times \dfrac{\mathrm{d}P}{\mathrm{d}t}$

$\dfrac{\mathrm{d}P}{\mathrm{d}t} = -\dfrac{135}{V^2} \times \dfrac{\mathrm{d}V}{\mathrm{d}t}$ OR $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -\dfrac{135}{P^2} \times \dfrac{\mathrm{d}P}{\mathrm{d}t}$ ***(A1)***

If $V = 20,\ \dfrac{\mathrm{d}V}{\mathrm{d}t} = -8$ (and $P = \dfrac{135}{20} = 6.75$)

$\dfrac{\mathrm{d}P}{\mathrm{d}t} = -\dfrac{-8(135)}{(20)^2} \left(= -\dfrac{-8(6.75)^2}{135} \right)$ ***(A1)***

$= 2.7\ \mathrm{Ncm^{-2}\ min^{-1}}$ ***A1***

METHOD 2

$PV = 135$

attempt to differentiate throughout with respect to t ***(M1)***

$P\dfrac{\mathrm{d}V}{\mathrm{d}t} + V\dfrac{\mathrm{d}P}{\mathrm{d}t} = 0$ ***(A1)***

If $V = 20,\ P = 6.75$ and $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -8$

$6.75(-8) + 20\dfrac{\mathrm{d}P}{\mathrm{d}t} = 0$ ***(A1)***

$\dfrac{\mathrm{d}P}{\mathrm{d}t} = 2.7\ \mathrm{Ncm^{-2}\ min^{-1}}$ ***A1***

Total [4 marks]

7. (a) recognition that total area $=1$ ***(M1)***

$\text{area of triangle (or sum of areas of triangles)} = 1$ OR $\int_0^5 f(x)\mathrm{d}x = 1$

$\frac{5b}{2} = 1$ ***A1***

Note: Award ***A1*** for a correct equation in b only.

$b = 0.4$ ***AG***

[2 marks]

(b) attempt to apply formula for $\mathrm{E}(X)$ to all pieces of the function ***(M1)***

$$\mathrm{E}(X) = \frac{b}{a}\int_0^a x^2\mathrm{d}x + \frac{b}{a-5}\int_a^5 x(x-5)\mathrm{d}x$$

$$= 0.4\left(\frac{1}{a}\int_0^a x^2\mathrm{d}x + \frac{1}{a-5}\int_a^5 (x^2-5x)\mathrm{d}x\right)$$ ***(A1)***

$$= 0.4\left(\frac{1}{a}\left[\frac{x^3}{3}\right]_0^a + \frac{1}{a-5}\left[\frac{x^3}{3} - \frac{5x^2}{2}\right]_a^5\right)$$

attempt to substitute limits, subtract in the correct order and equate to 3.2 ***(M1)***

$$0.4\left(\frac{1}{a}\left(\frac{a^3}{3}\right) + \frac{1}{a-5}\left[\left(\frac{5^3}{3} - \frac{5(5)^2}{2}\right) - \left(\frac{a^3}{3} - \frac{5a^2}{2}\right)\right]\right) = 3.2$$

$a = 4.6$ ***A1***

[4 marks]

Total [6 marks]

8. (a) attempt to find $\left(-2+2\mathrm{i}\sqrt{3}\right)^3$ OR $\left|-2+2\mathrm{i}\sqrt{3}\right|^3$ OR $\left(4\mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}\right)^3$ OR 4^3 ***(M1)***

Note: Award this ***M1*** for an attempt to cube $-2+2\mathrm{i}\sqrt{3}$ even if they have converted it incorrectly to exponential form for example $\left(4\mathrm{e}^{\frac{5\pi}{3}\mathrm{i}}\right)^3$.

$c=64$ ***A1***

[2 marks]

(b) $z=-2-2\mathrm{i}\sqrt{3}\ \ \left(=-2-3.46\mathrm{i}\right),\ \ z=4$ ***A1A1***

Note: Award ***A1*** for $-2-2\mathrm{i}\sqrt{3}$ only (no ***FT***), and ***A1(FT)*** for the real cube root of their c.
Accept $4+0\mathrm{i}$ instead of 4.

[2 marks]

(c) attempt to divide at least one solution by i OR find at least one cube root of their $c\mathrm{i}$ ***(M1)***

$z=\dfrac{-2\pm2\mathrm{i}\sqrt{3}}{\mathrm{i}}$, $z=\dfrac{4}{\mathrm{i}}$ OR $z^3=64\mathrm{i}\left(=64\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}\right)$

$z=-2\sqrt{3}+2\mathrm{i}\ \left(=-3.46+2\mathrm{i}\right),\ z=2\sqrt{3}+2\mathrm{i}\ \left(=3.46+2\mathrm{i}\right)\ ,\ \ z=-4\mathrm{i}$ ***A1***

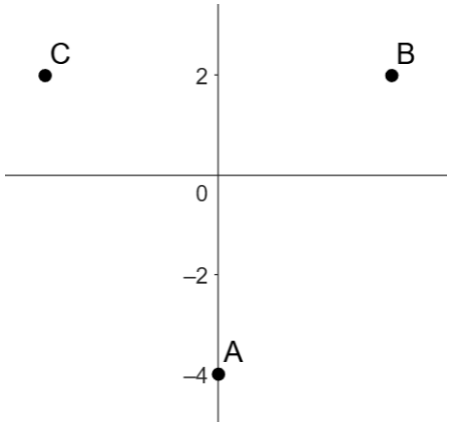
Note: Award ***M1A0*** for one or two correct answers, including those given in exponential form $4\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}, 4\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}, 4\mathrm{e}^{-\frac{\pi}{2}\mathrm{i}}$. Ignore incorrect or extra answers.

[2 marks]

continued…

Question 8 continued

(d)



valid approach to find area ***(M1)***

area $= 3\left(\frac{1}{2}(4)^2 \sin\frac{2\pi}{3}\right)$ OR $= \frac{1}{2}(6)\left(4\sqrt{3}\right)$ OR $= \frac{1}{2}\left(4\sqrt{3}\right)^2 \sin\frac{\pi}{3}$

$= 20.7846...$

$= 20.8\left(= 12\sqrt{3}\right)$ ***A1***

Note: Award ***FT*** from (c) provided their solutions are the cube roots of a number with modulus 64 (i.e. they have modulus 4 and form an equilateral triangle).

[2 marks]

Total [8 marks]

9. (a) attempt to make x the subject ***(M1)***

$x = d + \sqrt{4-(y-2)^2}$ or $x = d - \sqrt{4-(y-2)^2}$ ***A1***

Note: Award ***A1*** for either expression for x.

attempt to subtract two integrals of the form $\pi\int_0^4 x^2 \mathrm{d}y$ ***M1***

$V = \pi\int_0^4 \left(d + \sqrt{4-(y-2)^2}\right)^2 \mathrm{d}y - \pi\int_0^4 \left(d - \sqrt{4-(y-2)^2}\right)^2 \mathrm{d}y$ OR

$V = \pi\int_0^4 \left(\left(d + \sqrt{4-(y-2)^2}\right)^2 - \left(d - \sqrt{4-(y-2)^2}\right)^2\right)\mathrm{d}y$ ***A1***

attempt to expand brackets or use difference of two squares to simplify ***M1***

$V = \pi\int_0^4 \left(2d\sqrt{4-(y-2)^2} + 2d\sqrt{4-(y-2)^2}\right)\mathrm{d}y$ OR $V = \pi\int_0^4 \left(2d \times 2\sqrt{4-(y-2)^2}\right)\mathrm{d}y$ ***A1***

$V = 4\pi d\int_0^4 \sqrt{4-(y-2)^2}\,\mathrm{d}y$ ***AG***

[6 marks]

(b) $V = 4\pi d(6.28318...)\left(= 78.9568...\ d = 8\pi^2 d\right)$ ***(A1)***

$d = \dfrac{50\pi^2}{78.9568...}\left(= \dfrac{50\pi^2}{8\pi^2}\right)$

$= 6.25\left(= \dfrac{50}{8} = \dfrac{25}{4}\right)$ (exact answer only) ***A1***

[2 marks]

Total [8 marks]

Section B

10. (a) **EITHER**

attempt to use compound interest formula with $\text{PV} = 15000$, $r = 4.8$ and $k \neq 1$ ***(M1)***

$15000\left(1+\frac{4.8}{4(100)}\right)^{4(5)}\left(=15000(1.012)^{20}\right)$ ***(A1)***

OR

use of Finance App with correct PV and evidence of compounding >1 time per year ***(M1)***

$\text{N} = 5$		$\text{N} = 20$		$\text{N} = 20$	
$\text{I\%} = 4.8$		$\text{I\%} = 4.8$		$\text{I\%} = 1.2$	
$\text{PV} = -15000$		$\text{PV} = -15000$		$\text{PV} = -15000$	
$(\text{PMT} = 0)$	OR	$(\text{PMT} = 0)$	OR	$(\text{PMT} = 0)$	***(A1)***
$(\text{FV} =)$		$(\text{FV} =)$		$(\text{FV} =)$	
$\text{PpY}=1$		$\text{PpY}=4$		$\text{PpY}=1$	
$\text{CpY} = 4$		$\text{CpY} = 4$		$\text{CpY} = 1$	

Note: Accept use of $\text{PV} = 15000$ leading to $\text{FV} = -19041.51...$ up to this point.

THEN

$\text{FV} = 19041.51...$

$= \$19042$ ***A1***

Note: Final answer must be positive and given to the nearest whole dollar.

[3 marks]

continued…

Question 10 continued

(b) **METHOD 1**

EITHER

attempt to set up an equation with $\text{FV} = 28000,\ \text{PV} = 15000$ ***(M1)***

$28000 = 15000\left(1 + \frac{4.8}{4(100)}\right)^{4x}$ OR $28000 = 15000(1.012)^{x}$ ***(A1)***

OR

use of Finance App with correct PV and FV. ***(M1)***

$(\text{N} =)$		$(\text{N} =)$		$(\text{N} =)$	
$\text{I}\% = 4.8$		$\text{I}\% = 4.8$		$\text{I}\% = 1.2$	
$\text{PV} = -15000$		$\text{PV} = -15000$		$\text{PV} = -15000$	
$(\text{PMT} = 0)$	OR	$(\text{PMT} = 0)$	OR	$(\text{PMT} = 0)$	***(A1)***
$\text{FV} = 28000$		$\text{FV} = 28000$		$\text{FV} = 28000$	
$\text{PpY}=1$		$\text{PpY}=4$		$\text{PpY}=1$	
$\text{CpY} = 4$		$\text{CpY} = 4$		$\text{CpY} = 1$	

THEN

(number of years) $= 13.0810...$ OR (number of quarters) $= 52.3243...$ ***A1***

Lavinia will have enough money during the year 2039 ***AG***

[3 marks]

METHOD 2

attempt to use a table of values with $\text{PV} = 15000$ ***(M1)***

$\text{FV} = 15000\left(1 + \frac{4.8}{4(100)}\right)^{4x}$ OR $\text{FV} = 15000(1.012)^{x}$ ***(A1)***

after 13 years (start of 2039) $27891.88...$ AND after 14 years (start of 2040) $29254.99...$

OR 52 quarters $27891.88...$ AND 53 quarters $28226.59...$

OR 52 quarters $27891.88...$ AND 54 quarters $28565.30...$

OR 52 quarters $27891.88...$ AND 55 quarters $28908.09...$

OR 52 quarters $27891.88...$ AND 56 quarters $29254.99...$ ***A1***

Lavinia will have enough money during the year 2039 ***AG***

[3 marks]

continued…

Question 10 continued

(c) attempt to set up an equation or expression to find r ***(M1)***

$26222 = 28000\left(1 - \frac{r}{100}\right)$ OR $\frac{28000 - 26222}{28000}$

0.0635 (exact)

$r = 6.35$ ***A1***

[2 marks]

(d) value of the boat after x complete years = $28\,000\left(1 - \frac{6.35}{100}\right)^x$ ***(A1)***

equating their growth function from part (b) with their depreciation function ***(M1)***

$15\,000\left(1 + \frac{4.8}{4(100)}\right)^{4x} = 28\,000\left(1 - \frac{6.35}{100}\right)^x$ (accept inequality) ***(A1)***

EITHER

$x = 5.50788...$ ***(A1)***

OR

one row of values from above the thick line and one row from below the thick line ***(A1)***

x	LHS	RHS
5	19041.51...	20169.58...
5.25	19270.01...	19841.47...
5.5	19501.25...	19518.69...
5.75	19735.26...	19201.17...
6	19972.09...	18888.81...

THEN

Lavinia will buy the boat during the year 2031 ***A1***

[5 marks]

Total [13 marks]

11. (a) (normal vector to Π is) $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ (seen anywhere) ***(A1)***

attempt to set up equation of the form $(\boldsymbol{r}=)\boldsymbol{a}+\lambda\boldsymbol{d}$ using the normal vector ***(M1)***

$\boldsymbol{r}=\begin{pmatrix}4\\16\\-5\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ ***A1***

Note: Vector equation must include $\boldsymbol{r}=$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=$ for final ***A1***.

[3 marks]

(b) $\boldsymbol{r}=\begin{pmatrix}4+\lambda\\16+2\lambda\\-5-\lambda\end{pmatrix}$

attempt to substitute coordinates of a point on the line into the equation of Π ***(M1)***

$4+\lambda+2(16+2\lambda)-(-5-\lambda)=5(\Rightarrow 6\lambda+41=5)$

$\lambda=-6$ ***(A1)***

centre at $(-2,4,1)$ ***A1***

Note: Accept a position vector using any notation for final ***A1***.

[3 marks]

(c) (i) $\sqrt{(-3-(-2))^2+(6-4)^2+(4-1)^2}$ OR $\sqrt{1^2+2^2+3^2}$ ***A1***

$=\sqrt{14}\ \text{cm}^2$ ***AG***

(ii) curved surface area of hemisphere $=2\pi\left(\sqrt{14}\right)^2\left(=28\pi=87.9645...\right)$ (seen anywhere) ***(A1)***

attempt to find slant height of cone ***(M1)***

$\sqrt{(4-(-3))^2+(16-6)^2+(-5-4)^2}=\sqrt{7^2+10^2+9^2}$

slant height of cone $=\sqrt{230}\left(=15.1657...\right)$ ***(A1)***

curved surface area of cone $=\pi\sqrt{14}\sqrt{230}\left(=2\pi\sqrt{805}=178.269...\right)$ ***(A1)***

total surface area $=28\pi+2\pi\sqrt{805}=266.234...$

$=266\left(=\pi\left(2\sqrt{805}+28\right)\right)\ \text{cm}^2$ ***A1***

[6 marks]

continued…

Question 11 continued

(d) **EITHER**

attempt to find angle between $\pm\overrightarrow{VP}$ and $\pm\overrightarrow{VP'}$ using scalar product ***(M1)***

$\cos P\hat{V}P' = \dfrac{\begin{pmatrix}-7\\-10\\9\end{pmatrix}\cdot\begin{pmatrix}-13\\-5\\6\end{pmatrix}}{\sqrt{230}\sqrt{230}}$ OR $\cos P\hat{V}P' = \dfrac{\begin{pmatrix}7\\10\\-9\end{pmatrix}\cdot\begin{pmatrix}-13\\-5\\6\end{pmatrix}}{\sqrt{230}\sqrt{230}}$ or equivalent

OR

attempt to find PP' and the angle $P\hat{V}P'$ using cosine rule ***(M1)***

$PP' = \sqrt{(-9-(-3))^2+(11-6)^2+(1-4)^2} = \sqrt{70}$ and $\cos P\hat{V}P' = \dfrac{\sqrt{230}^2+\sqrt{230}^2-\sqrt{70}^2}{2\sqrt{230}^2}$

THEN

$\cos\theta = \dfrac{195}{\sqrt{230}\sqrt{230}}\left(=\dfrac{195}{230}=0.847826...\right)$ ***(A1)***

$\theta = 32.0239...^\circ$

$\theta = 32.0^\circ$ ***A1***

Note: Accept 32°.
Award ***(M1)(A1)A0*** for a radian answer of $\theta = 0.559$

[3 marks]

(e) attempt to find normal vector ***(M1)***

$\begin{pmatrix}-7\\-10\\9\end{pmatrix}\times\begin{pmatrix}-13\\-5\\6\end{pmatrix} = \begin{pmatrix}-60-(-45)\\-117-(-42)\\35-130\end{pmatrix}\left(=\begin{pmatrix}-15\\-75\\-95\end{pmatrix}=-5\begin{pmatrix}3\\15\\19\end{pmatrix}\right)$ ***(A1)***

attempt to substitute a point into their $ax+by+cz=d$ ***(M1)***

$3(-3)+15(6)+19(4)$ OR $3(-9)+15(11)+19(1)$ OR $3(4)+15(16)+19(-5)$

$3x+15y+19z=157$ (or equivalent such as $-15x-75y-95z=-785$) ***A1***

$a=3,\ b=15,\ c=19,\ d=157$

[4 marks]
Total [19 marks]

12. (a) (i) **METHOD 1**

finite limit if $\lim_{x\to -1}\left(x^2+ax+b\right)=0$ ***R1***

$\left(\lim_{x\to -1}\left(x^2+ax+b\right)=\right)1-a+b=0$ ***A1***

$b=a-1$ ***AG***

METHOD 2

recognition that $x+1$ is a factor of x^2+ax+b (so $x=-1$ is a zero) ***R1***

EITHER

$(-1)^2+(-1)a+b=0$ ***A1***

OR

$\frac{-a+\sqrt{a^2-4b}}{2}=-1$ AND $\sqrt{a^2-4b}=a-2$ AND $a^2-4b=a^2-4a+4$ ***A1***

OR

$x^2+ax+b=(x+1)(x+b)=x^2+(1+b)x+b$ AND $a=1+b$ ***A1***

THEN

$b=a-1$ ***AG***

Note: This ***A1*** is independent of the ***R1***.

METHOD 3

recognition that $x+1$ is a factor of x^2+ax+b (so remainder $=0$) ***(R1)***

$\frac{x^2+ax+b}{x+1}=x+(a-1)+\frac{b-a+1}{x+1}$ AND $b-a+1=0$ ***A1***

$b=a-1$ ***AG***

(ii) attempt to use l'Hôpital's rule or division ***(M1)***

$\lim_{x\to -1}\frac{x^2+ax+(a-1)}{x+1}=\lim_{x\to -1}\frac{2x+a}{1}$ OR $\frac{x^2+ax+(a-1)}{x+1}=x+(a-1)$

$\lim_{x\to -1}f(x)=a-2$ ***A1***

[4 marks]

(b) $x=-1$ ***A1***

[1 mark]

continued…

Question 12 continued

(c) **METHOD 1**

attempt at division by $x+1$ ***M1***

$\frac{x^2+ax+b}{x+1}=\frac{x(x+1)+(a-1)(x+1)+...}{x+1}$ OR long division

$\frac{x^2+ax+b}{x+1}=x+(a-1)+...$ ***A1***

oblique asymptote has equation $y=x+(a-1)$

$a-1=-4$ ***A1***

$a=-3$ ***AG***

METHOD 2

$\frac{x^2+ax+b}{x+1}=x-4+\frac{R}{x+1}$ (some constant R) ***(A1)***

$x^2+ax+b=(x-4)(x+1)+R$ ***A1***

$=x^2-3x+...$ ***A1***

$a=-3$ ***AG***

[3 marks]

(d) attempt to use quotient rule or product rule or chain rule ***(M1)***

$f'(x)=\frac{(x+1)(2x+a)-(x^2+ax+b)}{(x+1)^2}$ OR $-(x^2+ax+b)(x+1)^{-2}+(2x+a)(x+1)^{-1}$

OR $1-(b-a+1)(x+1)^{-2}$ OR equivalent with $a=-3$ ***A1A1***

Note: Award ***A1*** for $\frac{2x+a}{x+1}\left(=\frac{2x-3}{x+1}\right)$ and ***A1*** for $\frac{-(x^2+ax+b)}{(x+1)^2}\left(=\frac{-(x^2-3x+b)}{(x+1)^2}\right)$ from product and quotient rule.

Award ***A1*** for 1 and ***A1*** for $-(b-a+1)(x+1)^{-2}\left(=-(b+4)(x+1)^{-2}\right)$ from the chain rule.

$=\frac{x^2+2x-(b+3)}{x^2+2x+1}$

considering solutions of $x^2+2x-(b+3)=0\left(\Rightarrow (x+1)^2-(b+4)=0\right)$

continued…

Question 12 continued

EITHER

$2^2 + 4(b+3) \geq 0 \left(\Rightarrow 4b + 16 \geq 0\right)$ ***A1***

OR

$(x+1)^2 = b + 4$ ***A1***

THEN

$b \geq -4$ ***A1***

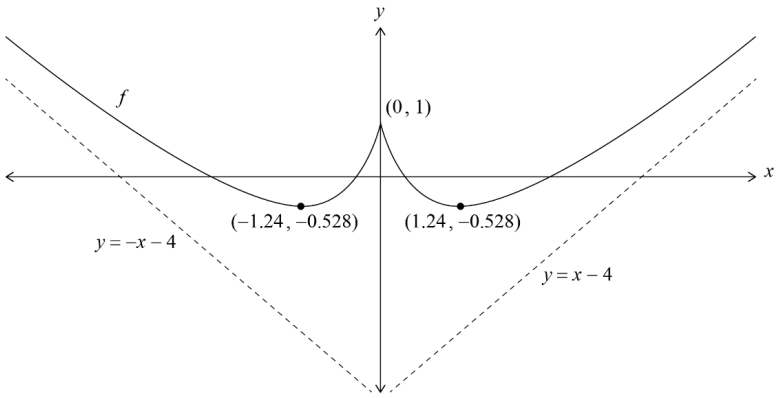
if $b = -4$ then $\lim_{x \to -1} f(x)$ is finite and no vertical asymptote (a straight line $y = x - 4$ with a discontinuity at $x = -1$) ***R1***

Note: Award the final ***A1R1*** if $(x+1)^2 = b + 4$ is seen together with $x \neq -1$ to give $b > -4$.

$b \geq -3$ ***AG***

[6 marks]

(e) (i)



smooth curve concave up for $x > 0$ with minimum below the x-axis ***A1***

correct coordinates for at least one minimum point ***A1***

symmetrical about the y-axis with cusp at $(0, 1)$ (accept 1 seen on axis) ***A1***

both asymptotes shown with correct equations ***A1***

appropriate asymptotic behaviour for both asymptotes ***A1***

(ii) $-1 < x < 0$ ***A1A1***

Note: Award ***A1A0*** for correct endpoints and incorrect inequality signs.

[7 marks]

Total [21 marks]